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Conserved Currents, Renormalization and Zero-Mass States.

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Summary. — It is shown that charge renormalization corresponds to a form of a spontaneous symmetry breakdown: although the charge operator exists, it can no longer be written as a space integral of the charge density. This can only happen due to the presence of zero-mass states in the theory, in full agreement with recent results on continuous symmetry groups in relativistic field theories.

1. – Introduction.

Recent papers have investigated the meaning of generators of continuous symmetry groups (generalized charges) as space integrals of densities in relativistic quantum field theories (1-3).

It has been shown that starting from a conserved current

$$\frac{\partial j^\mu(x)}{\partial x^\mu} = 0 \,, \tag{1}$$

associated to a symmetry (regarded as an invariance property of the equations of motion and commutation relations) one can, in the absence of zero-mass states, build a unitary operator

$$U(\tau) = \exp[i\tau Q] \,, \tag{2}$$

such that

$$U(\tau)\,|0\rangle = |0\rangle \,, \tag{3}$$

where $|0\rangle$ is the vacuum state.

(1) D. KASTLER, D. ROBINSON and J. A. SWIECA: *Commun. Math. Phys.*, **2**, 108 (1966).

(2) B. SCHROER and P. STICHEL: *Commun. Math. Phys.*, **3**, 208 (1966).

(3) J. A. SWIECA: *Phys. Rev. Lett.*, **17**, 974 (1966).

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Q is formally given by

$$Q = \int j^0(\boldsymbol{x}, t)\, \mathrm{d}^3x\,, \tag{4}$$

where eq. (4) should be understood to mean

$$\langle\psi|Q|\varphi\rangle = \lim_{v\to\infty} \langle\psi|\int_v j^0(\boldsymbol{x}, t)\, \mathrm{d}^3x\, |\varphi\rangle \tag{5}$$

with $|\psi\rangle, |\varphi\rangle$ normalized states out of a dense set in the Hilbert space.

When there are zero-mass states in the theory the arguments leading to (2), (3) and (5) fail and in particular there might be no operator Q. This corresponds to what is called a «spontaneous breakdown of symmetry» (4).

However, even when zero-mass states are present, one can still have theories without spontaneously broken symmetries, that is $U(\tau)$ satisfying (2), (3) exists and implements the symmetry but eq. (5) might no longer hold for $|\psi\rangle, |\varphi\rangle$ physically relevant states.

We shall show that this is precisely what happens in quantum electrodynamics corresponding to the effect of charge renormalization. We shall also prove that for massive vector mesons coupled to conserved currents such an effect does not occur in accordance with general results obtained in (1,2).

Another example of the failure of eq. (5) will be given in the case of the electron gas. There, the long-range forces in the interaction cause the effect that in a relativistic field theory is due to zero-mass states (5-7).

We shall therefore see that zero-mass states (or long-range forces in a many-body system), besides providing a mechanism for a possible spontaneous breakdown of symmetry, can also lead to a weaker form of breakdown. The symmetry is still a good one, but the usual connection between its generator and the space integral of the density no longer holds.

2. – Results valid in the absence of zero-mass states.

We shall summarize here some of the results which follow from (1,2,8), for internal continuous symmetry groups in theories with a nonzero lowest mass.

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(4) J. Goldstone, A. Salam and S. Weinberg: *Phys. Rev.*, **127**, 965 (1962); Y. Nambu and G. Jona Lasinio: *Phys. Rev.*, **122**, 345 (1961); J. Goldstone: *Nuovo Cimento*, **19**, 154 (1961).

(5) R. V. Lange: *Phys. Rev. Lett.*, **14**, 3 (1965).

(6) G. S. Guralnik, T. Kibble and C. R. Hagen: *Phys. Rev. Lett.*, **13**, 585 (1964).

(7) J. A. Swieca: *Commun. Math. Phys.*, **4**, 1 (1967).

(8) H. Ezawa and J. A. Swieca: *Commun. Math. Phys.*, **5**, 330 (1967).

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Introducing local operators A as Wightman polynomials [9,10] in the basic fields $\varphi_i(x)$ smeared out with functions with compact support

$$A = \sum_{n=0}^{N} \int f_n(x_1 \dots x_n) \varphi_{i_1}(x_1) \dots \varphi_{i_n}(x_n)\, \mathrm{d}^4x_1 \dots \mathrm{d}^4x_n\,, \tag{6}$$

one has the symmetry at its most basic level as a correspondence (automorphism)

$$A \to A_\tau\,, \tag{7}$$

which leaves the equations of motion and commutation relations invariant.

The connection between (7) and the conserved current (1) is given by

$$\left.\frac{\mathrm{d}A_\tau}{\mathrm{d}\tau}\right|_{\tau=0} = i\,[j^0(f_R f_d), A]\,, \qquad R > R_0\,, \tag{8}$$

with

$$j^0(f_R f_d) = \int j^0(x)\, f_R(\boldsymbol{x})\, f_d(x_0)\, \mathrm{d}^4x\,, \tag{9}$$

with f_R, f_d smooth functions satisfying

$$\begin{cases} f_R(\boldsymbol{x}) = 1\,, & |\boldsymbol{x}| < R\,, \\ f_R(\boldsymbol{x}) = 0\,, & |\boldsymbol{x}| > R + \varepsilon\,, \end{cases} \tag{10}$$

$$\begin{cases} f_d(x_0) = 0\,, & |x_0| > d\,, \\ \int f_d(x_0)\, \mathrm{d}x_0 = 1\,, \end{cases} \tag{11}$$

and R_0 being such that the points

$$(x_0, \boldsymbol{x}), \text{ with } \begin{cases} |x_0| < d\,, \\ |\boldsymbol{x}| > R_0\,, \end{cases} \tag{12}$$

lie out of the light cone of the finite space-time region to which A is associated.

Equation (8) is a more careful way of writing

$$\left.\frac{\mathrm{d}A_\tau}{\mathrm{d}\tau}\right|_{\tau=0} = i\left[\int_v j^0(\boldsymbol{x}, t)\, \mathrm{d}^3x, A\right], \qquad v > v_0\,, \tag{13}$$

([9]) R. Streater and A. Wightman: *PCT, Spin, Statistics and All That* (New York, 1964).

([10]) R. Haag: *Phys. Rev.*, **112**, 669 (1958).

which in turn would follow from *formally* writing (4),

$$A_\tau = \exp[i\tau Q]\, A \exp[-i\tau Q]\,, \tag{14a}$$

$$\left.\frac{\mathrm{d}A_\tau}{\mathrm{d}\tau}\right|_{\tau=0} = i[Q, A]\,, \tag{14b}$$

and using local commutativity. Since one really wants to show that Q exists it is eq. (8) that ought to be taken as a starting point and not eqs. (4), (14).

One can now *define* an operator Q acting on the dense set of states obtained by applying local operators on the vacuum (local states) by

$$QA|0\rangle \stackrel{\text{def}}{=} [j^0(f_R f_d), A]|0\rangle\,, \qquad R > R_0\,. \tag{15}$$

The Q so defined is unique and Hermitian as a consequence of (cf. [1,4,8])

$$\langle 0|[j^0(f_R f_d), A]|0\rangle = 0\,, \qquad R > R_0\,. \tag{16}$$

One can then proceed to build $U(\tau) = \exp[i\tau Q]$ by a power series expansion which converges on local states (for internal symmetries) obtaining thus a time-independent unitary operator that satisfies

$$U(\tau)|0\rangle = |0\rangle\,, \tag{17a}$$

$$U(\tau)A|0\rangle = A_\tau|0\rangle\,, \tag{17b}$$

$$U(\tau)AU^{-1}(\tau) = A_\tau \text{ (on local states)}\,. \tag{17c}$$

The transformation properties under the symmetry group of the asymptotic in and out states, the multiplet structure etc., follow immediately from (17*b*) by using the Haag-Ruelle [10,11] asymptotic-state construction.

Furthermore the formal relation (4) is to be replaced by [1,2]

$$\langle\psi|Q|\varphi\rangle = \lim_{R\to\infty}\langle\psi|j^0(f_R f_d)|\varphi\rangle\,, \tag{18}$$

with $|\varphi\rangle$, $|\psi\rangle$ states out of the dense set obtained by applying Wightman polynomials with fast decreasing smearing functions on the vacuum (quasi-local states).

Since in the absence of zero-mass states one can obtain any normalized one-particle state by applying a local operator smeared out with a fast decreasing function on the vacuum [10,11]

$$|1\rangle = \int f(x)A(x)\,\mathrm{d}^4x|0\rangle\,, \tag{19}$$

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[11] D. RUELLE: *Helv. Phys. Acta*, **35**, 147 (1962).

with $A(x)$ the translate of A by x, one has

$$\langle 1|Q|1\rangle = \lim_{R\to\infty} \langle 1|j^0(f_R f_d)|1\rangle . \tag{20}$$

Equation (20) constitutes the proof of the well-known statement that coupling constants attached to conserved currents do not get renormalized [12], when no zero-mass states are present.

3. – The case of zero-mass states.

When there are zero-mass states which are connected to the vacuum state by the current operator, the techniques employed in [1,2,8] leading to eqs. (17), (18), (20) no longer apply.

If the symmetry, however, is not spontaneously broken a «charge» operator will still exist and satisfy eqs. (17) and also

$$[Q, A] = [j^0(f_R f_d), A] , \qquad R > R_0 . \tag{21}$$

It remains to be seen now what happens to eqs. (18), (20). By using quantum electrodynamics as an example it will be shown that eq. (20) is no longer valid due to vacuum polarization effects which lead to a charge renormalization. Arguments will be given later in this Section to support our view that eq. (18) is still valid, and a general proof presented in Appendix B.

We shall work with covariant quantum electrodynamics in the Gupta-Bleuler gauge, and rather freely manipulate with renormalization constants as if they were finite.

Taking the equation for the unrenormalized photon field as

$$\Box A^\mu(x) = e_0 j^\mu(x) , \tag{22a}$$

where $j^\mu(x)$ satisfies the conservation law (1) and is formally given by

$$j^\mu(x) = [\bar{\psi}(x), \gamma^\mu \psi(x)]/2 , \tag{22b}$$

one finds using equal-time commutation relations

$$[j^0(f_R f_d), \ \psi(x)] = \psi(x) , \qquad R > R_0 , \tag{23a}$$

$$[j^0(f_R f_d), A^\mu(x)] = 0 , \qquad R > R_0 . \tag{23b}$$

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(12) R. P. Feynman and M. Gell-Mann: *Phys. Rev.*, **109**, 193 (1958).

It might happen that such a formal use of equal-time commutators leads to wrong results and that a more careful definition of the current operators will show that eqs. (23) do not hold. Such a possibility was first pointed out by JOHNSON in his analysis of the Thirring model ([13]). However if one wants to maintain the usual form of the generalized Ward identity ([14]) one needs (23). This point has been recently clarified by BARDAKCI, HALPERN and SEGRÉ ([15]) (*).

Since the charge is a good quantum number in q.e.d. there exists an operator Q such that

$$(24a)\qquad [Q, \psi(x)] = \psi(x)\,,$$

$$(24b)\qquad [Q, A^\mu(x)] = 0$$

and

$$(25)\qquad \begin{cases} Q|0\rangle = 0\,, \\ Q|1\rangle = |1\rangle\,, \end{cases}$$

where $|1\rangle$ is a normalized one-electron state.

On the other hand

$$(26)\qquad \lim_{R\to\infty} \langle 1|j^0(f_R f_d)|1\rangle \doteq \lim_{\tilde f_R(\,)\to\delta^3(p)} \int \psi_i^*(\boldsymbol{p}')\, \tilde f_R(\boldsymbol{p}'-\boldsymbol{p})\cdot\langle p_i'|j^0|p_j\rangle\cdot\psi_j(\boldsymbol{p})\,\mathrm{d}^3p\,\mathrm{d}^3p'\,,$$

where $\sim$ indicates the Fourier transform, $\psi_j(\boldsymbol{p})$ is the wave function in momentum spin space and the form factor is given by the Feynman diagram analysis

$$(27)\qquad \langle p_i'|j^\mu|p_j\rangle = \bar u_i(\boldsymbol{p}')\left\{\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\frac{1}{1-\pi(k^2)} + \frac{k^\mu k^\nu}{k^2}\right\}\Gamma_r^\nu(p', p)\,u_j(\boldsymbol{p})\,,$$

with

$$k = (p' - p)\,,$$

and Γ_r^ν, the renormalized proper vertex, satisfies on account of the Ward identity ([14])

$$(28a)\qquad \Gamma_r^\mu(p', p) = \gamma^\mu\,, \qquad \begin{cases} \gamma\cdot p = m\,, \\ p' = p\,, \end{cases}$$

$$(28b)\qquad k^\mu \Gamma_r^\mu(p', p) = 0\,, \qquad \gamma\cdot p = \gamma\cdot p' = m.$$

([13]) K. JOHNSON: *Nuovo Cimento*, **20**, 773 (1961).

([14]) J. C. WARD: *Phys. Rev.*, **77**, 293 (1950); Y. TAKAHASHI: *Nuovo Cimento*, **6**, 370 (1957).

([15]) K. BARDAKCI, M. B. HALPERN and G. SEGRÉ: preprint.

(*) I am grateful to Prof. Th. MARIS for calling my attention to their paper.

Therefore (26), (27), (28) lead to

$$(29)\qquad \lim_{R\to\infty}\langle 1|j^0(f_R f_d)|1\rangle = \frac{1}{1-\pi(0)} = Z_3 \neq \langle 1|Q|1\rangle = 1\,.$$

The fact that the one-electron matrix elements of the charge operator Q do not coincide with the matrix elements of the space integral of the charge density although they lead to the same commutators, is due to the well-known phenomenon of vacuum polarization [16]. Part of the charge goes to infinity and is never picked up by integrating the density over arbitrary large volumes.

An argument now will be represented in favor of the validity of eq. (18) although eq. (20) obviously fails. This will mean that it is due to the impossibility of building a one-particle state by means of (19) with $f(x)$ a fast decreasing function (since there is no mass gap between the one-electron hyperboloid and the continuum of scattering states) that eq. (20) breaks down.

We shall discuss this in the related but simpler problem of polarization by an external weak charge. The induced charge density in the vacuum is given by

$$(30)\qquad \langle j^0(x)\rangle_{\text{ind}} = \int\langle 0|[j^0(x), A^0(x')]_{\text{Ret}}|0\rangle\, e_0\varrho(\boldsymbol{x}')\,\mathrm{d}^4x'$$

and the induced total charge

$$(31)\qquad \lim_{v\to\infty}\int_v\langle j^0(x)\rangle_{\text{ind}}\,\mathrm{d}^3x = \lim_{\substack{k\to 0\\ k_0=0}}\frac{\pi(k^2)}{1-\pi(k^2)}\,\varrho(\boldsymbol{k}) = -(1-Z_3)\int\varrho(\boldsymbol{x})\,\mathrm{d}^3x\,,$$

which as in (29) means that a fraction $(1-Z_3)$ of the charge goes to infinity. If we imagine the external charge to be switched on at a finite time $-T$ and off at some time prior to 0, then eq. (30) can also be written as

$$(32)\qquad \langle j^0(\boldsymbol{x}, 0)\rangle_{\text{ind}} = \int\langle 0|[j^0(\boldsymbol{x}, 0), A^0(x')]|0\rangle\cdot$$
$$\cdot\, e_0\varrho(\boldsymbol{x}')g(x^{0\prime})\,\mathrm{d}^4x' = \langle 0|[j_0(\boldsymbol{x}, 0), A^0(e_0\varrho g)]|0\rangle\,,$$

where g is a function with compact support. With (23) and (25) one finds that the total induced charge in this case is zero

$$(33)\qquad \lim_{v\to\infty}\int_v\langle j^0(\boldsymbol{x}, 0)\rangle_{\text{ind}}\,\mathrm{d}^3x = \langle 0|[Q_v, A^0(e_0\varrho g)]|0\rangle = 0\,, \qquad v > v_0\,.$$

([16]) J. Schwinger: *Phys. Rev.*, **76**, 790 (1949).

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Equation (33) can be checked directly by using the Lehman-Källén [17] representation for the commutator

$$(34)\qquad \langle 0|[j^\mu(x), A^\nu(0)]|0\rangle = \frac{-1}{(2\pi)^3}\int\Big[(1-Z_3)k^\mu k^\nu\delta(k^2) + \\ + \sigma(k^2)[g^{\mu\nu}k^2 - k^\mu k^\nu]\exp[-ikx]\varepsilon(k_0)\Big]\mathrm{d}^4k$$

leading to

$$(35)\qquad \lim_{v\to\infty}\int_v \langle j^0(\boldsymbol{x},0)\rangle_{\text{ind}}\,\mathrm{d}^3x = \lim_{\boldsymbol{k}\to 0}\Big[-\int\tilde{g}(k_0)\,\tilde{\varrho}(\boldsymbol{k})\cdot \\ \cdot[(1-Z_3)k^{02}\delta(k^2) + \sigma(k^2)(k^2-k^{02})]\mathrm{d}k_0\varepsilon(k_0)\Big] = 0\,,$$

which is zero for $\tilde{g}(k_0)$ a smooth and fast decreasing function. In the static limit $\tilde{g}(k_0)\to 1/(k_0 - i\varepsilon)$ there will be a nonvanishing contribution from the $\delta(k^2)$ term

$$(36)\qquad \lim_{\boldsymbol{k}\to 0}\Big[-\int\frac{\tilde{\varrho}(k)}{k_0 - i\varepsilon}\,k_0^2(1-Z_3)\delta(k^2)\varepsilon(k_0)\mathrm{d}k_0\Big] = (Z_3 - 1)\int\varrho(\boldsymbol{x})\,\mathrm{d}^3x\,,$$

in accordance with (31).

The situation just described means that it takes an infinite time to induce a nonzero total charge in the vacuum.

The analogy with eqs. (18), (20) is clear: eq. (18) corresponds to the case of an external charge acting during a finite time and should still be valid; eq. (20) corresponds to the static limit and as shown in eq. (29) breaks down.

It is perhaps useful to comment at this stage that the fact that a fraction $(1-Z_3)$ of the bare charge goes to infinity is in no contradiction with the usual charge renormalization

$$(37)\qquad e = Z_3^{\frac{1}{2}}e_0\,.$$

This can be seen from dual role of the charge as a quantum number and coupling constant. While as a quantum number the charge is reduced by a factor Z_3, the way to measure electric charge is by its role as a coupling constant. In this case one is measuring the interaction energy which, as for the case of a classical dielectric (cf. Appendix A), is reduced by an amount Z_3 and therefore as a coupling constant the charge gets renormalized according to (37).

The physical charge operator is now conveniently defined as

$$(38)\qquad Q_p = eQ\,,$$

[17] G. KÄLLEN: *Helv. Phys. Acta*, **25**, 417 (1952); H. LEHMANN: *Nuovo Cimento*, **11**, 342 (1954).

satisfying

$$\langle 1|Q_v|1\rangle = e = \lim_{v\to\infty} \langle 1|eZ_3^{-1}\int_v j^0(\boldsymbol{x}, t)\,\mathrm{d}^3x|1\rangle\,, \tag{39}$$

whereas formally

$$Q_v = e\int j^0(\boldsymbol{x}, t)\,\mathrm{d}^3x\,. \tag{40}$$

Equation (40) is consistent as long as commutators of Q_v with local operators are concerned, but cannot be used for the calculation of particle matrix elements.

We finally remark that the general statements of Sect. **2** can be directly verified if, instead of q.e.d., one takes a theory of massive vector mesons coupled to conserved currents with no zero-mass bound states.

$$(\Box + \mu_0^2)\varphi^\mu(x) = g_0 j^\mu(x)\,. \tag{41}$$

In this case, instead of (26), (27) one has

$$\lim_{R\to\infty} \langle 1|j^0(f_R f_d)|1\rangle = \lim_{\tilde{f}(\boldsymbol{p})\to\delta^3(\boldsymbol{p})} \int \psi_i^*(\boldsymbol{p}')\,\tilde{f}_R(\boldsymbol{p}'-\boldsymbol{p})\cdot \cdot\,\overline{u}_i(\boldsymbol{p}')\,\frac{k^2-\mu_0^2}{k^2-\mu_0^2-k^2\pi(k^2)}\,\Gamma_r^0\,(p'p)\,u_j(\boldsymbol{p})\,\psi_j(\boldsymbol{p})\,\mathrm{d}^3p\,\mathrm{d}^3p'\,, \tag{42}$$

which leads to

$$\lim_{R\to\infty} \langle 1|j^0(f_R f_d)|1\rangle = \langle 1|Q|1\rangle = 1\,, \tag{43}$$

since in the absence of zero-mass states $\Gamma_r^\mu(p'p)$ is not singular at $k^2\to 0$ and the usual arguments leading to Ward's identity (28*a*), (28*b*) apply.

4. – The electron gas.

An even more drastic example of the failure of a symbolic formula like (4) in the calculation of relevant matrix elements is provided by the electron gas. There the one « extra » electron matrix element of the integral of the density gives (for instance in the random-phase approximation cf. [18])

$$\lim_{v\to\infty}\langle 1|e\int_v \varrho(\boldsymbol{x}, t)\,\mathrm{d}^3x|1\rangle = \lim_{k,w\to 0} \frac{e\boldsymbol{k}^2}{k^2+\pi(\boldsymbol{k}, w)}\,. \tag{44}$$

([18]) D. Pines: *Elementary Excitations in Solids* (New York, 1963).

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Since

(45) $$\lim_{k,w\to 0} \pi(\boldsymbol{k}, w) = k_T^2 ,$$

with k_T the Fermi-Thomas screening vector, one obtains

(46) $$\lim_{v\to\infty} \langle 1|\int_v \varrho(\boldsymbol{x}, t)\, \mathrm{d}^3x|1\rangle = 0 .$$

Equation (46) means that the electron gas tends to (locally) neutralize any extra charge, giving rise to screening. Despite eq. (46) the invariance of the Wightman functions [9] of the theory

(47) $$W(x_1 \dots x_m\, y_1 \dots y_n) = \langle 0|\psi^+(x_1) \dots \psi^+(x_m)\psi(y_1) \dots \psi(y_m)|0\rangle ,$$

with $|0\rangle$ the ground state of the system, under the gauge transformation

(48) $$\psi(x) \to \exp[ie\tau]\psi(x) ,$$

implies [9,1] the existence of a charge operator Q_v with

(49) $$[Q_v, \psi(x)] = e\left[\int_v \varrho(\boldsymbol{x}, t)\, \mathrm{d}^3x,\, \psi(\boldsymbol{x}, t)\right], \qquad v > v_0 ,$$

and

(50) $$Q_v|0\rangle = 0 ,$$

which is then a measure of the number of « extra » electrons in the gas.

Appendix A

It is illustrative to see the differences between the case of zero-mass states (or long-range forces) and massive states (short-range forces) already in the classical situation of charges in an infinitely extended dielectric medium.

The Poisson equation for the static potential φ is

(A.1) $$\nabla^2\varphi = -e_0(\varrho_{\text{ext}} + \varrho_{\text{ind}})$$

and, with

(A.2) $$\varrho_{\text{ind}} = -\eta\, \mathrm{div}\, \boldsymbol{E} = \eta\nabla^2\varphi ,$$

leads to the solution

(A.3) $$\varphi(\boldsymbol{x}) = \frac{1}{4\pi}\frac{e_0}{1 + e_0\eta}\int \frac{\varrho_{\text{ext}}(\boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|}\, \mathrm{d}^3x'$$

and the total charge is

$$e_0\int(\varrho_{\text{ext}}+\varrho_{\text{ind}})\,\mathrm{d}^3x=\frac{e_0}{1+e_0\eta}\int\varrho_{\text{ext}}(x')\,\mathrm{d}^3x'\,, \tag{A.4}$$

where $(1+e_0\eta)^{-1}$ is the analog of Z_3 in (29).

The total energy in the dielectric is given by

$$H=\frac{e_0}{2}\int\varrho_{\text{ext}}(\boldsymbol{x}')\varphi(\boldsymbol{x}')\,\mathrm{d}^3x'=\frac{e_0^2}{1+e_0\eta}\frac{1}{8\pi}\int\varrho_{\text{ext}}(\boldsymbol{x}'')\frac{1}{|\boldsymbol{x}'-\boldsymbol{x}''|}\varrho_{\text{ext}}(\boldsymbol{x}'')\,\mathrm{d}^3x'\,\mathrm{d}^3x''\,. \tag{A.5}$$

This leads to a coupling constant renormalization

$$e=\frac{e_0}{(1+e_0\eta)^{\frac{1}{2}}}\,, \tag{A.6}$$

although the total charge (A.4) was reduced by a factor $(1+e_0\eta)^{-1}$.

If one substitutes the Coulomb law by a Yukawa one

$$(\nabla^2-\mu_0^2)\,\varphi=-g_0(\varrho_{\text{ext}}+\varrho_{\text{ind}})\,, \tag{A.7}$$

with (A.2) one obtains

$$\varphi(\boldsymbol{x})=\frac{g_0}{4\pi(1+g_0\eta)}\int\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}\exp\left[-\frac{\mu_0}{(1+\eta g_0)^{\frac{1}{2}}}|\boldsymbol{x}-\boldsymbol{x}'|\right]\cdot\varrho_{\text{ext}}(\boldsymbol{x}')\,\mathrm{d}^3x'\,, \tag{A.8}$$

which leads to a mass and conventional coupling constant renormalization

$$g=g_0(1+\eta g_0)^{-\frac{1}{2}}\,,\qquad \mu=\mu_0(1+\eta g_0)^{-\frac{1}{2}}\,. \tag{A.9}$$

However the total «charge» is not renormalized

$$g_0\int(\varrho_{\text{ext}}+\varrho_{\text{ind}})\,\mathrm{d}^3x=g_0\int\varrho_{\text{ext}}\,\mathrm{d}^3x\,, \tag{A.10}$$

in analogy with (43).

Appendix B

We shall outline here the proof that, in a relativistic theory, if the symmetry is not spontaneously broken, eq. (18) holds even in the presence of zero-mass states. With (21) and (17*a*)

$$\langle 0|[j^0(f_R f_d),A]|0\rangle=\langle 0|[Q,A]|0\rangle=0\,,\qquad R>R_0\,, \tag{B.1}$$

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and this implies (cf. [8])

$$\text{(B.2)}\qquad \langle 0|[j^0(x), A]|0\rangle = \int_0^\infty \mathrm{d}\varkappa^2 \int \nabla\boldsymbol{\sigma}_1(\varkappa^2, \boldsymbol{y})\, \Delta(x-y, \varkappa^2)\, \mathrm{d}^3y + \\ + \int_0^\infty \mathrm{d}\varkappa^2 \int \nabla\boldsymbol{\sigma}_2(\varkappa^2, \boldsymbol{y})\, \frac{\partial}{\partial t}\Delta(x-\boldsymbol{y}, \varkappa^2)\, \mathrm{d}^3y\,,$$

with $\sigma_{12}(\varkappa^2, \boldsymbol{y})$ being of compact support in the variable $\boldsymbol{y}$ due to the locality of A.

Equation (B.2) gives

$$\text{(B.3)}\qquad \lim_{R\to\infty} \langle 0|j^0(f_R f_d) A|0\rangle = \lim_{\tilde{f}_R(\boldsymbol{p})\to\delta^3(\boldsymbol{p})} \Big[-i\int_0^\infty \mathrm{d}\varkappa^2 \int \mathrm{d}^3p\, \cdot \\ \cdot \frac{\tilde{f}_d\big(\sqrt{\boldsymbol{p}^2+\varkappa^2}\big)}{\sqrt{\boldsymbol{p}^2+\varkappa^2}}\, f_R(\boldsymbol{p})\boldsymbol{p}\cdot\tilde{\boldsymbol{\sigma}}_1(\varkappa^2, \boldsymbol{p}) - \int_0^\infty \mathrm{d}\varkappa^2 \int \mathrm{d}^3p \cdot \tilde{f}_d\big(\sqrt{\boldsymbol{p}^2+\varkappa^2}\big)\tilde{f}_R(\boldsymbol{p})\boldsymbol{p}\cdot\tilde{\boldsymbol{\sigma}}_2(\varkappa^2, \boldsymbol{p})\Big]\,.$$

Since $\tilde{\boldsymbol{\sigma}}_{1,2}(\varkappa^2, \boldsymbol{p})$ are analytic in $\boldsymbol{p}$ being Fourier transforms of functions with compact support and $\tilde{f}_d\big(\sqrt{\boldsymbol{p}^2+\varkappa^2}\big)$ is a fast decreasing function providing a high mass cut-off the only possible cause for the nonvanishing of (B.3) is the zero-mass contribution to $\tilde{\boldsymbol{\sigma}}_1$.

Taking

$$\text{(B.4)}\qquad \tilde{\boldsymbol{\sigma}}_1(\varkappa^2, 0) = \boldsymbol{l}\delta(\varkappa^2)$$

one indeed would have a result for (B.3) that depends on the way the function $f_R(\boldsymbol{x})$ tends to unity when $R\to\infty$.

The ansatz (B.4) is however incompatible with the locality of A since for zero-mass states

$$\text{(B.5)}\qquad \langle 0|j^0|p\rangle = \lambda p^0 = \lambda|\boldsymbol{p}|\,,$$

which means with (B.4)

$$\text{(B.6)}\qquad \langle p|A|0\rangle \simeq \boldsymbol{l}\cdot\boldsymbol{p}/|\boldsymbol{p}|\,, \qquad \boldsymbol{p}\sim 0\,.$$

Considering the commutator of A^+ and A, locality requires [19]

$$\text{(B.7)}\qquad |\langle p|A|0\rangle|^2 = a(\boldsymbol{p}) + |\boldsymbol{p}|b(\boldsymbol{p})\,,$$

with a and b analytic functions of $\boldsymbol{p}$. This is incompatible with (B.6) and (B.4) unless $\boldsymbol{l} = 0$.

([19]) H. ARAKI, K. HEPP and D. RUELLE: *Helv. Phys. Acta*, **35**, 164 (1962).

In order to have compativility between (B.2), (B.5) and (B.7) one must have

$$\langle p|A|0\rangle \sim \beta(\boldsymbol{p}) , \qquad \varkappa, \boldsymbol{p}\sim 0 , \tag{B.8}$$

and the zero-mass contribution to $\tilde{\boldsymbol{\sigma}}_1$

$$\tilde{\boldsymbol{\sigma}}_1(\varkappa^2, \boldsymbol{p}) \sim \boldsymbol{p}\delta(\varkappa^2) , \qquad \varkappa^2 \sim 0,\ \boldsymbol{p}\sim 0 . \tag{B.9}$$

Inserting (B.9) into (B.3) one realizes that also the zero-mass terms go to zero in the limit so that

$$\lim_{R\to\infty} \langle 0|j^0(f_R f_d) A|0\rangle = 0 . \tag{B.10}$$

Using (17), (21) we obtain

$$\lim_{R\to\infty} \langle 0|Bj^0(f_R f_d) A|0\rangle = \langle 0|BQA|0\rangle , \tag{B.11}$$

for A, B local operators, completing the proof for local states.

It is easy to extend this result to quasi-local states.

RIASSUNTO (*)

Si fa vedere come la rinormalizzazione di carica corrisponde ad una forma di rottura spontanea della simmetria: sebbene l'operatore di carica esista, non può essere ancora scritto come un integrale spaziale della densità di carica. Questo può accadere solo per la presenza nella teoria di stati di massa nulla, in pieno accordo coi recenti risultati sui gruppi continui di simmetria ottenuti nelle teorie di campo relativistiche.

(*) *Traduzione a cura della Redazione.*

Сохраняющиеся токи, перенормировка и состояния с нулевой массой.

Резюме (*). — Показывается, что перенормировка заряда соответствует форме спонтанного нарушения симметрии: хотя оператор заряда существует, но он не может больше быть записан, как пространственный интеграл от плотности заряда. Это может произойти только из-за наличия состояний с нулевой массой в теории, что находится в полном согласии с недавними результатами в непрерывных группах симметрий в релятивистских теориях поля.

(*) *Переведено редакцией.*